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# Tuning of Passivity-Preserving Controllers for Switched-Mode Power Converters

Dimitri Jeltsema and Jacquélien M. A. Scherpen

**Abstract**—Nonlinear passivity-based control (PBC) algorithms for power converters have proved to be an interesting alternative to other, mostly linear, control techniques. The control objective is usually achieved through an energy reshaping process and by injecting damping to modify the dissipation structure of the system. However, a key question that arises during the implementation of the controller is how to tune the various control parameters. From a circuit theoretic perspective, a PBC forces the closed-loop dynamics to behave as if there are artificial resistors—the control parameters—connected in series or in parallel to the real circuit elements. In this paper, a solution to the tuning problem is proposed that uses the classical Brayton–Moser equations. The method is based on the study of a certain “mixed-potential function” which results in quantitative restrictions on the control parameters. These restrictions seem to be practically relevant in terms stability, overshoot and nonoscillatory responses. The theory is exemplified using the elementary single-switch buck and boost converters.

**Index Terms**—Brayton–Moser equations, controller commissioning, passivity-based control, power converters, tuning.

## I. INTRODUCTION

IN RECENT years passivity-based control (PBC) design for switched-mode power converters has become quite an active area in both the field of system and control theory and power electronics. One particular PBC technique is based on the classical Euler–Lagrange (EL) equations. The application of EL-based PBC design to single-switch dc-to-dc power converters was first proposed by Sira-Ramírez *et al.* [13] and is generalized to larger networks, like the coupled-inductor Cuk converter, and three-phase rectifiers and inverters in e.g., [3], [7], and [12]. One of the major advantages of using the EL approach is that the physical structure (e.g., energy, dissipation, and interconnection), including the nonlinear phenomena and features, is explicitly incorporated in the model, and thus in the corresponding PBC. This in contrast to conventional techniques that are mainly based on linearized dynamics and corresponding proportional-integral-derivative (PID) or lead-lag control. Since many power converters are nonlinear nonminimum phase systems, controllers stemming from linear techniques are sometimes difficult to tune as to ensure robust performance, especially in the presence of large setpoint changes and disturbances that cause circuit operation to deviate from the nominal point of operation. Therefore,

incorporating knowledge about the nonlinear dynamics in the controller design may be beneficial.

The basic idea behind PBC design is to modify the energy of the system and add damping by modification of the dissipation structure. In the context of EL-based PBC designs for power-converters, two fundamental questions arise.

- 1) Which variables have to be stabilized to a certain value in order to regulate the output(s) of interest toward a desired equilibrium value? In other words, are the zero-dynamics of the output(s) to be controlled stable<sup>1</sup> with respect to the available control input(s), and if not, for which state variables is it stable?
- 2) Where to inject the damping and how to tune the various parameters associated to the energy modification and to the damping assignment stage?

It is hard to give a general answer to the first question since we are not able to give explicit formulations of the zero-dynamics for a general converter structure. Application of PBC to, for example, the boost, buck-boost [13] and the Ćuk [12] converter, leads to an indirect regulation scheme of the output voltage through regulation of the input current. Since there is no general answer to the first question yet, we continue with checking the stability of the zero-dynamics on a case by case basis.

A first attempt to develop some guidelines for adjusting the damping parameters is done by studying the disturbance attenuation properties and look for upper and lower bounds on these parameters using  $\mathcal{L}_2$ -gain analysis techniques in [11]. Since the  $\mathcal{L}_2$ -gain analysis can be argued to be intrinsically conservative and, in case of large converter structures, the necessary calculations may become rather complex, we study a more practical approach. To our knowledge, apart from  $\mathcal{L}_2$ -gain analysis, there are some recent interesting works revealing Hamiltonian-based results related to tuning (see, for instance, [9]). However, Hamiltonian-based PBCs differ from EL-based PBCs, as considered herein.

In previous works about EL-based PBC, the location where to add the damping is mainly motivated by the form of the open-loop dissipation structure in the sense that damping is added to those states that do not contain any damping terms *a priori*. For example, for the boost converter this means that only damping is injected on the input current—called *series* damping—because the output voltage already contains a damping term due to

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<sup>1</sup>We should emphasize that in general PBC is a technique that aims at energy shaping and *not* at imposing any specific behaviors to certain signals (i.e., outputs to be controlled). However, in some applications (including a large class of power-converters), EL-based PBC designs may lead to a partial system inversion. In that case, a thorough study of the zero-dynamics becomes a subproduct of the method. For a detailed discussion, see [7].

the load resistance, e.g., [7] and [13]. The latter control scheme leads to a PBC regulated circuit that is highly sensitive to load variations and also needs an expensive current sensor to measure the inductor current. This holds for many other switching networks too. Recently, in [3] we have proposed a preliminary solution to overcome the load sensitivity problem by using the concept of *parallel* damping injection.

In this paper, we further develop the preliminary results of [3] and [5] by adopting the Brayton–Moser (BM) equations [1] to analyze a closed-loop PBC scheme. The main contributions can be summarized as follows: In Section II, we briefly introduce the BM equations and accommodate them for the inclusion of controlled switches. The first part of Section III presents the reformulation of the EL-based PBC design procedure in terms of the BM equations, which will be referred to as BM-based PBC. Since PBC design is based on modification of the physical structure of the circuit, it is not surprising that the BM framework allows an interpretation of the closed-loop dynamics in similar physical terms, i.e., in terms of inductors, capacitors and resistors. From a circuit-theoretic point of view, the PBC produces a control signal which forces the closed-loop dynamics to act as if there are *virtual* resistors connected in series and/or in parallel to the real circuit elements [3], [5]. The second part of Section III presents quantitative guidelines concerning the adjustments of the control parameters based on modified versions of the stability theorems proposed in [1]. Additionally, the latter theorems justify the possibility to choose either a series or a parallel damping injection scheme. Illustrative examples using the elementary buck and boost converters, which describe in form and function a large family of power converters, are presented in Sections IV and V. Based on these two case studies, some novel aspects concerning the robustness properties of the parallel damping injection schemes stemming from BM-based PBC are suggested.

## II. SWITCHED BM EQUATIONS

### A. Non-Switched Electrical Circuits

Consider an electrical circuit with  $\sigma$  inductors and  $\rho$  capacitors. Assume that there are no capacitor-only loops and no inductor-only cutsets. In the early sixties, BM have shown [1] that the dynamical behavior of a broad class<sup>2</sup> of nonlinear electrical circuits is governed by the following differential equations:

$$\begin{aligned} -L(i_L)\frac{di_L}{dt} &= \frac{\partial P}{\partial i_L}(i_L, v_C) \\ C(v_C)\frac{dv_C}{dt} &= \frac{\partial P}{\partial v_C}(i_L, v_C) \end{aligned} \quad (1)$$

where  $i_L = \text{col}(i_{L_1}, \dots, i_{L_\sigma}) \in \mathbb{R}^\sigma$  denote the currents through the  $\sigma$  inductors, and  $v_C = \text{col}(v_{C_1}, \dots, v_{C_\rho}) \in \mathbb{R}^\rho$  denote the voltages across the  $\rho$  capacitors, respectively, and  $P : \mathbb{R}^{\sigma \times \rho} \rightarrow \mathbb{R}$  is called the *mixed-potential* function, to be specified later. The matrices  $L(i_L) \in \mathbb{R}^{\sigma \times \sigma}$  and  $C(v_C) \in \mathbb{R}^{\rho \times \rho}$  denote the inductance and the capacitance matrices, respectively. Notice

<sup>2</sup>This class covers all topologically complete electrical circuits. A circuit is called “topologically complete” if it can be described by an independent set of inductor currents and capacitor voltages such that Kirchhoff’s laws are satisfied. For a detailed treatment, the reader is referred to [16].

that the first equation of (1) constitutes Kirchhoff’s voltage law, while the second constitutes Kirchhoff’s current law. Furthermore, as argued in [6], (1) do not establish a Lagrangian system in the classical sense, but they can be viewed as some *degenerate* Lagrangian form.

The mixed-potential function captures the interconnection structure, dissipation structure and external signals. A simple procedure to obtain  $P(i_L, v_C)$  can be summarized as follows.

- First, treat all series resistors and voltage sources as a short circuit, and treat all parallel resistors and current sources as an open circuit. Apply either Kirchhoff’s current law or Kirchhoff’s voltage law to the remaining circuit, i.e., determine  $i_C = -\Psi^T i_L$  or  $v_L = \Psi v_C$ , respectively, where  $\Psi$  denotes a matrix of appropriate dimensions.
- The internal power circulating across the dynamic elements is represented by  $P_T(i_L, v_C) = i_L^T \Psi v_C$ .
- Determine the dissipative current-potential  $P_R(i_L)$  that captures the influence of the current-controlled resistors  $R$ , i.e.,

$$P_R(i_L) = \int_0^{i_L} v_R(i'_L) di'_L.$$

- Determine the dissipative voltage-potential  $P_G(v_C)$  that captures the influence of the voltage-controlled resistors (conductors)  $G$ , i.e.,

$$P_G(v_C) = \int_0^{v_C} i_G(v'_C) dv'_C.$$

- The total supplied power by the (current-controlled) voltage sources  $E$  and the (voltage-controlled) current sources  $J$  are represented by  $P_E(i_L)$  and  $P_J(v_C)$ , respectively.
- Finally, the mixed-potential  $P(i_L, v_C)$  is determined by combining the potentials obtained in the previous steps as

$$P(i_L, v_C) = P_T(i_L, v_C) + P_R(i_L) - P_G(v_C) - P_E(i_L) + P_J(v_C).$$

For ease of notation, in the sequel we will use the more compact notation

$$Q(x)\dot{x} = \frac{\partial P}{\partial x}(x) \quad (2)$$

where  $x = \text{col}(i_L, v_C)$  and  $Q(x) = \text{diag}(-L(i_L), C(v_C))$ .

*Remark 1:* In contrast to the Lagrangian or Hamiltonian functions, that are usually defined by the circuit’s total (co-)energy, the mixed-potential consists of terms related to the power circulating in the circuit. However, it is easily seen that the circuit’s total co-energy, denoted by  $H^*(i_L, v_C)$ , forms a fundamental part of the BM equations, e.g., [4], i.e., we may replace  $L(i_L)$  and  $C(v_C)$  in (1) by

$$L(i_L) = \frac{\partial^2 H^*}{\partial i_L^2}(i_L, v_C) \quad C(v_C) = \frac{\partial^2 H^*}{\partial v_C^2}(i_L, v_C)$$

respectively. This property is of main importance in the following section. Other similarities and dualities between the BM equations and port-Hamiltonian systems can be found in [4].

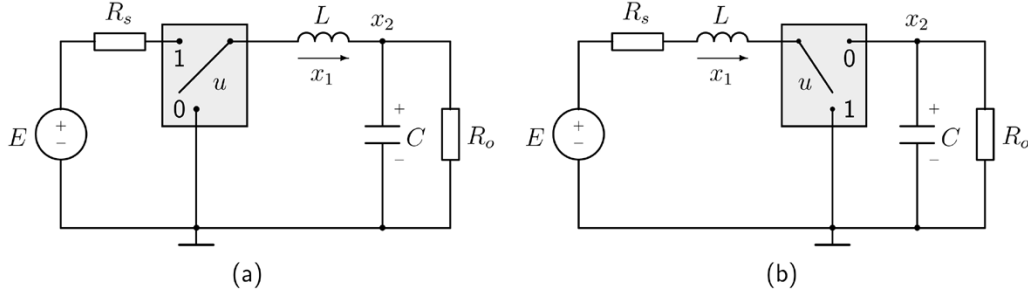


Fig. 1. Examples. (a) Buck type converter. (b) Boost type converter.

*Remark 2:* The resistive current and voltage potentials,  $P_R(i_L)$  and  $P_G(v_C)$ , are often referred to as the resistive content and co-content, respectively. This terminology was introduced by Millar in the early 1950s; see [1] and the references therein. In case of linear resistors, the content is simply half the dissipated power expressed in terms of the inductor currents, while the co-content is half the dissipated power in terms of the capacitor voltages. On the other hand, from an EL perspective  $P_R(i_L)$  and  $P_G(v_C)$  may be considered as some generalized Rayleigh dissipation functions.

### B. Switched-Mode Electrical Circuits

For circuits that contain one or more switches, we denote the switch position(s) by  $u = (u_1, \dots, u_m)$ , where  $u_j \in U \triangleq \{0, 1\}$ ,  $j = 1, \dots, m$ , i.e., ON or OFF, or in other words  $u$  is in the discrete set  $U^m$ . Depending on the application, redefinition of the switching function may also result in, for example,  $u_j \in U \triangleq \{-1, 0, 1\}$ . The mixed-potential function is modified to include switching functions  $u$  by letting  $P(x) = P^u(x)$ . For circuits containing a single switch  $P^u(x)$  is defined as

$$P_\gamma^u(x) \triangleq uP_\gamma^1(x) + (1-u)P_\gamma^0(x) \quad (3)$$

with  $\gamma \in \{T, R, G, E, J\}$ ,  $P_\gamma^1(x)$  is the mixed-potential function for the switch position  $u = 1$ , and  $P_\gamma^0(x)$  is the mixed-potential function for the switch position  $u = 0$ . The way the switch enters the potential function as defined in (3) differs from the definition of the switched Rayleigh dissipation function, as defined in [3], [7], and [12], in the sense that here we have used the concept of superposition of the power flows, where in the references the switch in the dissipation would enter via the dissipated energy. Although we have modified the mixed-potential function for the inclusion of one controllable switch, the approach is also suitable for circuits with more than one switch. Inspired by [7], the dynamics of a switched circuit are then expressed by means of a quadruple  $\Sigma^u$ , called the *switched BM parameters*

$$\Sigma^u = \{H^*(x), P_T^u(x), P_D^u(x), P_F^u(x)\}$$

where  $P_D^u(x) = P_R^u(i_L) + P_G^u(v_C)$  and  $P_F^u(x) = P_E^u(i_L) + P_J^u(v_C)$ . Consequently, for every admissible switch vector  $u \in U$  we have a different but unique set of parameters  $\Sigma^u$ .

Let us next consider the BM dynamics of the single-switch buck and boost converters. These converters describe in form and function a large family of power converters and, therefore, we will use them to exemplify the theory throughout this paper.

*1) Buck Converter:* Consider the buck converter depicted in Fig. 1(a), where we have defined  $x_1 = i_L$  and  $x_2 = v_C$ . If we assume that all elements are linear and time-invariant,  $H^*(x) = (1/2)Lx_1^2 + (1/2)Cx_2^2$ , or  $Q = \text{diag}(-L, C)$ . The internal potential  $P_T^u(x)$  is readily found as  $P_T^u(x) = x_1x_2$ . Furthermore, the dissipative current and voltage potentials are  $P_R^u(x_1) = (1/2)uR_sx_1^2$ ,  $P_G^u(x_2) = (1/2)G_ox_2^2$ , where  $G_o = 1/R_o$  represents the load conductance. The supplied power by the voltage source equals  $P_E^u(x_1) = uEx_1$  and  $P_J^u(x_2) = 0$ . Notice that for this circuit only  $P_E^u(x_1)$  depends on the position of the switch. The equations describing the dynamical behavior of the buck type converter are then given by

$$\begin{aligned} -L\dot{x}_1 &= \frac{\partial P^u}{\partial x_1}(x_1, x_2) = x_2 + u(R_sx_1 - E) \\ C\dot{x}_2 &= \frac{\partial P^u}{\partial x_2}(x_1, x_2) = x_1 - G_ox_2. \end{aligned} \quad (4)$$

*2) Boost Converter:* The BM parameters of the boost converter, depicted in Fig. 1(b), are exactly the same as in the previous case, except for the fact that the internal power becomes a function of the switch position  $u$ , i.e.,  $P_T^u(x) = (1-u)x_1x_2$ , and the current potentials now equal  $P_R^u(x_1) = (1/2)R_sx_1^2$  and  $P_E^u(x_1) = Ex_1$ . The resulting equations for the boost converter are then given by

$$\begin{aligned} -L\dot{x}_1 &= \frac{\partial P^u}{\partial x_1}(x_1, x_2) = (1-u)x_2 + R_sx_1 - E \\ C\dot{x}_2 &= \frac{\partial P^u}{\partial x_2}(x_1, x_2) = (1-u)x_1 - G_ox_2. \end{aligned} \quad (5)$$

### C. Pulse-Width Modulation

The switched BM equations are closely related to the average pulse-width modulation (PWM) models, under the condition that the PWM frequency is sufficiently high; see [7] for a detailed discussion in the EL context. This means that  $x$  is replaced by the average state  $z$ , representing the average inductor currents and capacitor voltages, and the discrete control  $u$  is replaced by its duty ratio function vector  $\mu$ . For circuits containing a single switch, we thus have the following consistency conditions

$$P_\gamma^\mu(z)|_{\mu=1} = P_\gamma^1(z) \quad \text{and} \quad P_\gamma^\mu(z)|_{\mu=0} = P_\gamma^0(z).$$

The averaged potentials  $P_\gamma^\mu(z)$  can be considered as a weighted ratio, with weighting parameter  $\mu$ , between  $P_\gamma^1(z)$  and  $P_\gamma^0(z)$ .

In the sequel, we will use the average models with  $z$  denoting the vector of average inductor currents and capacitor voltages,

respectively, and  $\mu$  as the duty ratio of the switch, operating in the closed interval  $[0, 1]$ .

### III. BM-BASED PASSIVITY-BASED CONTROL

We are now ready to apply passivity based control to the average PWM BM models. As explained in Subsection II-C, the state vector  $x$  is replaced by its averaged value  $z$ , representing the average inductor currents and capacitor voltages, and the discrete control  $u$  is replaced by its duty ratio function vector  $\mu$ , i.e., we consider switched BM models of the form

$$Q(z)\dot{z} - \frac{\partial P^\mu}{\partial z}(z) = 0. \quad (6)$$

Furthermore, without loss of generality, we assume in our developments that the external voltage and current sources are known and constant.

#### A. Passivity-Based Controller Design

The rationale behind the design of a passivity-based controller for switched-mode circuits is to modify the closed-loop co-energy and add damping by modification of the dissipative structure; see [7]. This means that we start by modifying the (average) co-energy function  $H^*(z)$  to arrive at a desired closed-loop co-energy function and modify the (average) dissipative potential  $P_D^\mu(z)$ . To do this, let  $\tilde{z} \triangleq z - \xi$  define the average state errors, where  $\xi$  represents the desired trajectories for the average inductor currents and capacitor voltages, respectively. Furthermore, if we take as desired closed-loop co-energy function  $H_M^*(z, \xi) \triangleq H^*(z)|_{z=\tilde{z}}$ , the design procedure of the PBC reduces to first, making a copy of system (6) in terms of  $\xi$ , and second, by adding damping in the errors to ensure asymptotic stability, i.e.,

$$Q(\xi)\dot{\xi} - \frac{\partial P^\mu}{\partial \xi}(\xi) + \Lambda \frac{\partial P_V^\mu}{\partial \xi}(z - \xi) = 0 \quad (7)$$

where  $P_V^\mu(z - \xi) = P_V^\mu(\tilde{z})$  is the injected dissipation and  $\Lambda \triangleq \text{diag}(-I_{\sigma \times \sigma}, I_{\rho \times \rho})$ . The resulting closed-loop dissipative potential evaluated in the error states is then defined by

$$P_M^\mu(\tilde{z}) \triangleq P_D^\mu(z)|_{z=\tilde{z}} + P_V^\mu(\tilde{z}). \quad (8)$$

Finally, an explicit definition of the control action is obtained after solving (7) for  $\mu$ . Due to the underlying partial system inversion,  $\mu$  needs to be solved with respect to a minimum phase state (or states) as discussed in the introduction. This is tantamount to setting the minimum phase states to be controlled to their desired values, and solve for the control  $\mu$  with respect to the remaining states. Hence, if we use  $\tilde{z} = z - \xi$  in (6) together with (7), and  $H_M^*(\tilde{z})$  and  $P_D^\mu(z)$  are quadratic in  $\tilde{z}$  and  $z$ , respectively, then the closed-loop error dynamics are in BM form again, i.e.,

$$Q\dot{\tilde{z}} - \frac{\partial P_M^\mu}{\partial \tilde{z}}(\tilde{z}) - \Lambda \frac{\partial P_M^\mu}{\partial \tilde{z}}(\tilde{z}) = 0. \quad (9)$$

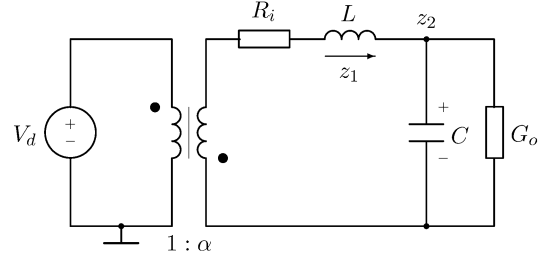


Fig. 2. Closed-loop interpretation of the series damping PBC regulated buck converter. The ideal transformer is included to compensate for the “virtual” voltage drop over  $R_i$ .

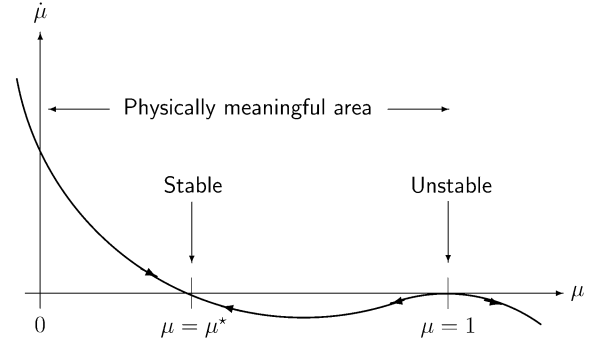


Fig. 3. Zero-dynamics for the parallel damping PBC controlled boost converter.

Invoking Lyapunov/LaSalle arguments, asymptotic stability of the closed-loop error dynamics (9) is easily checked by noting that

$$\dot{H}_M^*(\tilde{z}) = -S_M^\mu(\tilde{z}) \leq 0 \quad (10)$$

where  $S_M^\mu(\tilde{z}) \geq 0$  represents the total dissipated power of the closed-loop error system, and where the largest invariant set is given by  $\tilde{z} = 0$ . Notice that in case the closed-loop dissipation is linear,  $S_M^\mu(\tilde{z}) = 2P_M^\mu(\tilde{z})$  (see Remark 2).

#### B. Tuning of the PBC

So far, we have derived the procedure to obtain a PBC strategy in terms of the BM equations, as is developed in [7] based on the EL equations. We note that the design procedure in terms of the BM equations yields exactly the *same* controllers as one would obtain using an EL description. However, using the BM formulation, the controller is now directly expressed in physically measurable quantities, i.e., currents and/or voltages, while in the latter frameworks the controller is expressed in terms of charges and/or fluxes. Furthermore, it will be shown that the present setting provides us a systemic tool for tuning the PBC controllers.

Interestingly enough, in [1, Ths. 3 and 4, pp. 19 and 21], stability criteria are developed that use the mixed-potential function. These criteria can be used to rule out the existence of self-sustained oscillations. Hence, if we translate the ideas of [1] to our closed-loop setting, where we assume that the closed-loop error system is in BM form (9), we have strong criteria to tune the various control parameters. In other words, we can assign values to the injected dissipation functions to assure a desired

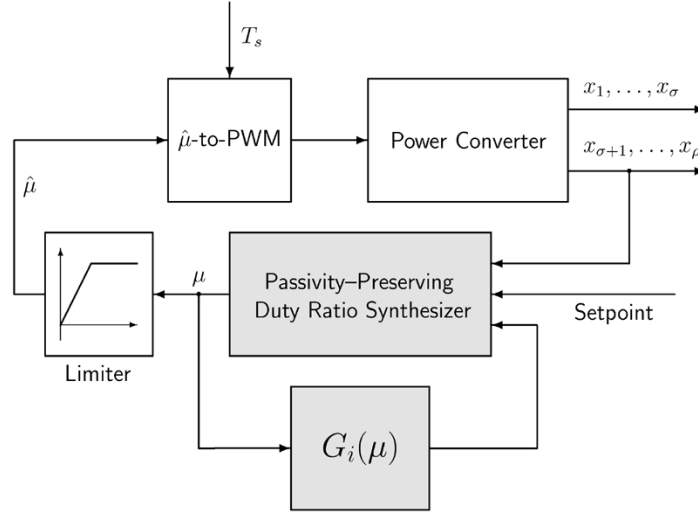


Fig. 4. General feedback representation of a parallel damping PBC regulated power converter.

dynamic behavior in terms of, for example, overshoot and robustness against load variations.

For the closed-loop error mixed-potential function, a qualitative Lyapunov-based stability condition for the system (9) is stated as follows. Let  $\tilde{z} = \text{col}(\tilde{v}, \tilde{w})$ , where  $\tilde{v} = \text{col}(\tilde{z}_1, \dots, \tilde{z}_\sigma)$  and  $\tilde{w} = \text{col}(\tilde{z}_{\sigma+1}, \dots, \tilde{z}_{\sigma+\rho})$ , denote the error-currents through the inductors and error-voltages across the capacitors. Furthermore, let

$$R(\tilde{v}) = \frac{\partial^2 P_M^\mu}{\partial \tilde{v}^2}(\tilde{z}) \quad G(\tilde{w}) = \frac{\partial^2 P_M^\mu}{\partial \tilde{w}^2}(\tilde{z})$$

denote the (modified) closed-loop resistance and conductance matrices, respectively.

**Theorem 1:** If  $R$  is a positive-definite constant matrix, and

$$\|K_R\| \triangleq \left\| L^{\frac{1}{2}} R^{-1} \Psi(\mu) C^{-\frac{1}{2}} \right\| = 1 - \gamma \quad (11)$$

with  $0 < \gamma < 1$ , then for all  $(\tilde{v}, \tilde{w})$  the solutions of (9) tend to zero as  $t \rightarrow \infty$ .<sup>3</sup>

**Remark 3:** Of course, in the previous subsection we already concluded that if  $P_M^\mu(\tilde{z}) \geq 0$  the closed-loop error dynamics converge to zero according to (10). However, Theorem 1 (and also Theorem 2, as stated later) forms a somewhat more conservative condition to ensure convergence of (9). Moreover, the theorem provides a lower bound on the control parameters to ensure a “reasonably nice” response in terms of, e.g., overshoot, settling-time, etc. To illustrate this point, consider the linearization of (9) in the vicinity of the equilibrium point  $\tilde{z} = \tilde{z}^* = 0$  and  $\mu = \mu^*$ , i.e.,  $\delta \dot{\tilde{z}} = A \delta \tilde{z}$ , where  $A$  denotes the linearized system matrix. Based on the linearized system it can be shown, in terms of the complex frequency domain, that if one of the theorems is satisfied, each eigenvalue of  $A$  lies either on the real axis (away from the origin) or on a circle in the left-half plane. The radius of this circle can be made arbitrarily small with  $\gamma$ . The interested reader is referred to [1] for a detailed discussion of this fact.

<sup>3</sup>Here, the notation  $\|K\|$  denotes the norm of  $K$ , defined as  $\|K\|^2 = \max_{\|x\|=1} \{[Kx]^\top Kx\}$ .

Although it is assumed that  $R$  is constant in the first place, the criterion of Theorem 1 places a constraint on  $R$  in terms of  $L, C$ , and  $\Psi(\mu)$ . Therefore, if  $\Psi(\mu)$  is not constant, it may be desirable to choose  $R$  as a function of  $\mu$  in order to fulfill (11). Notice that if Theorem 1 is satisfied, stability is guaranteed regardless of  $G(\tilde{w})$ ! A similar criterion for the  $G$ -matrix can be stated as follows.

**Theorem 2:** If  $G$  is a positive-definite constant matrix, and

$$\|K_G\| \triangleq \left\| C^{\frac{1}{2}} G^{-1} \Psi^T(\mu) L^{-\frac{1}{2}} \right\| = 1 - \gamma \quad (12)$$

with  $0 < \gamma < 1$ , then for all  $(\tilde{v}, \tilde{w})$  the solutions of (9) tend to zero as  $t \rightarrow \infty$ .

Detailed proofs for the case when  $\Psi$  is a constant matrix are given in [1]. The proofs for  $\Psi(\mu)|_{\mu=1}$  and  $\Psi(\mu)|_{\mu=0}$ , follow in a similar way. Stability of  $\Psi(\mu)$  for any admissible  $\mu$  follows from the fact that  $\Psi(\mu)$  satisfies the Lipschitz condition since  $\mu$  is bounded, see also [5]. Notice that  $\gamma$  may be considered as a fine-tuning parameter. The practical relevance of the criteria is illustrated in the following section (see Remark 5), where particular choices of  $\gamma$  appear to coincide with tuning criteria stemming from linear techniques, as proposed in [3] and [5].

### C. Series/Parallel Damping Injection

Apart from the qualitative behavior of the closed-loop system, the criteria of Theorem 1 and Theorem 2 enable us to choose between two different damping injection strategies: Theorem 1 suggests to add damping at all the inductor currents by injecting *series* resistances, while the criterion of Theorem 2 suggests to inject damping at the capacitor voltages by injecting *parallel* conductances, i.e., according to the theorems it is sufficient to modify either  $P_R^\mu(z)$  or  $P_G^\mu(z)$  by letting

$$P_M^\mu(\tilde{z}) = P_D^\mu(z)|_{z=\tilde{z}} + P_V^\mu(\tilde{v}) \quad (13)$$

or

$$P_M^\mu(\tilde{z}) = P_D^\mu(z)|_{z=\tilde{z}} + P_V^\mu(\tilde{w}) \quad (14)$$

respectively. Hence, stability is guaranteed by selecting either *series damping* injection (13) or *parallel damping* injection

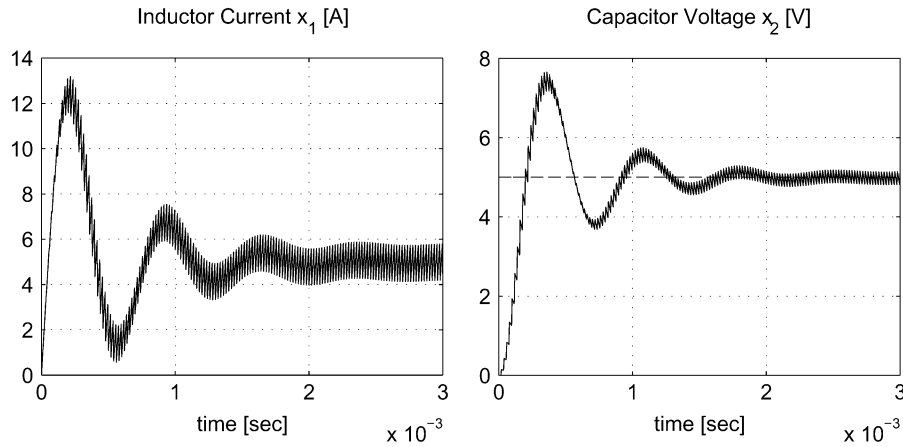


Fig. 5. Typical open-loop start-up response for  $V_d = 5V$ .

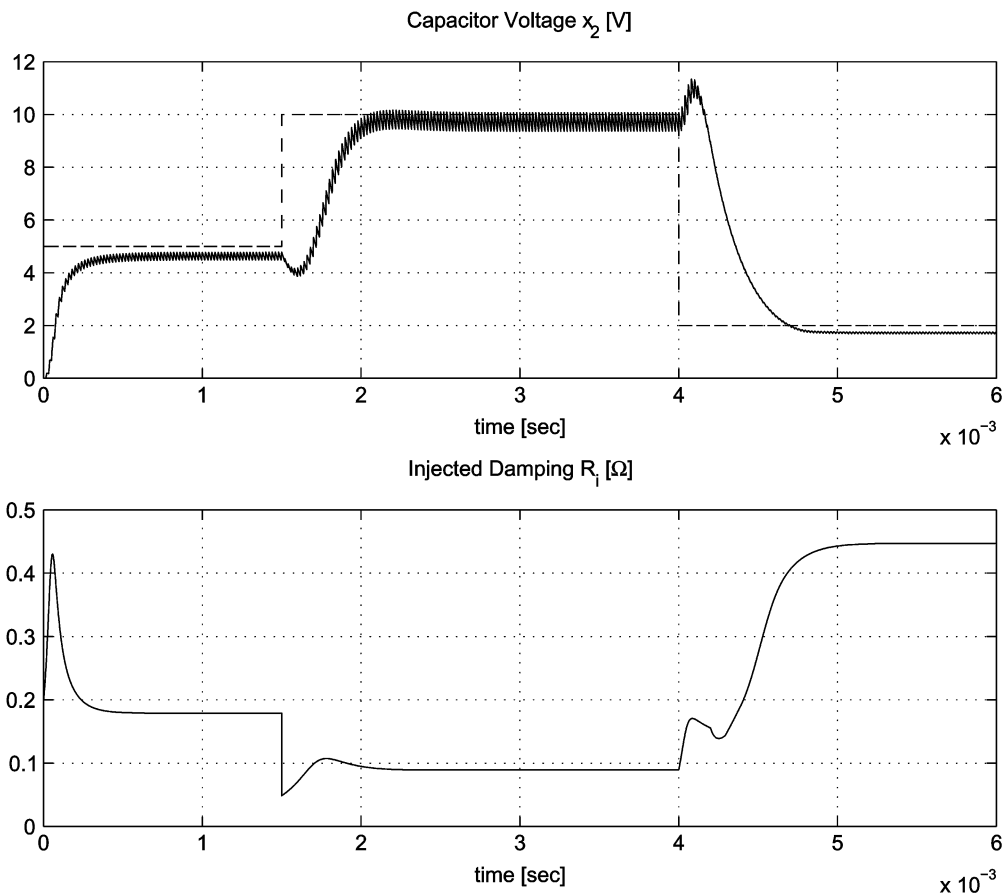


Fig. 6. Closed-loop response for different setpoints  $V_d$  based on series damping: capacitor voltage response (top); average injected damping  $R_i(\mu)$  (bottom).

(14), to satisfy the criteria of Theorems 1 or 2 for some suitable  $\gamma$ , respectively. The concept of parallel damping, as it follows from Theorem 2, coincides with the ideas as recently proposed in [3] and [5]. We will come back to this later on.

#### IV. TUNING EXAMPLES OF SERIES DAMPED PBC REGULATED CONVERTERS

In this section, we consider two illustrative examples of the series damping PBC strategy of Theorem 1. First, we treat an

example where the interconnection matrix  $\Psi$  is constant using the buck converter. Second, a series damping PBC for the boost converter is developed and its tuning rules are derived. For both converters we assume for simplicity (and without loss of generality) that the source resistance  $R_s = 0$ . Again, we point out that the main reason for studying these two converters is that they describe in form and function a large family of power converter structures. A complicating property of the boost converter, as for many more converter structures, is that due to the nonlinear behavior of the conversion ratio the converter's natural resonance frequency is varying with the

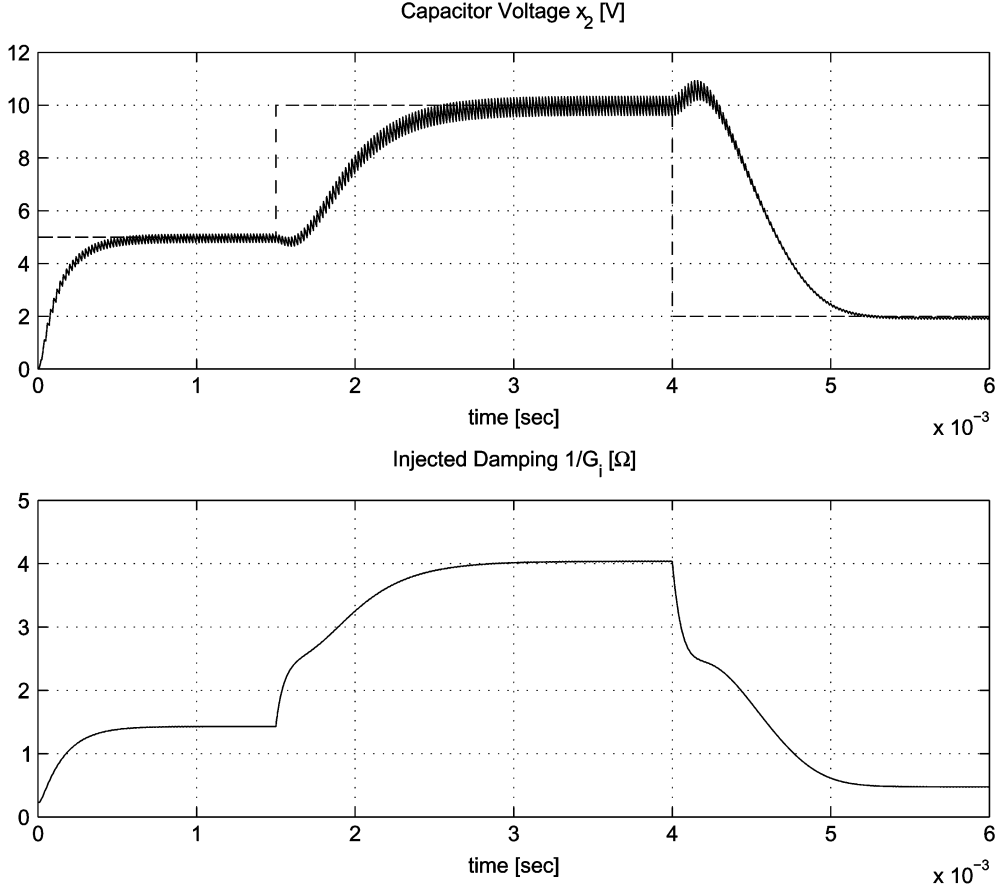


Fig. 7. Closed-loop response for different setpoints  $V_d$  based on parallel damping: capacitor voltage response (top); average injected damping  $1/G_i(\mu)$  (bottom).

desired output voltage<sup>4</sup>. This means that the tuning criterion will also depend on the desired output voltage.

#### A. Buck Converter

Consider the average BM dynamics of the PWM controlled buck converter (4) derived in Section II. In average mode the state is represented by  $z$  and the control by  $\mu$ . The control objective is to regulate the capacitor voltage  $z_2$  toward a desired value, say  $z_2 = V_d$ , without any overshoot. Furthermore, we first assume that  $G_o$  is perfectly known. As demonstrated in [13], both direct and indirect regulation is possible because the zero-dynamics for both  $z_1$  and  $z_2$ , with respect to the control  $\mu$ , are stable. Suppose we set  $P_V^\mu(z_1, \xi_1) = (1/2)R_i(z_1 - \xi_1)^2$  (series damping injection), then the closed-loop error dynamics satisfy

$$\begin{aligned} L\dot{\tilde{z}}_1 &= -R_i\tilde{z}_1 - \tilde{z}_2 \\ C\dot{\tilde{z}}_2 &= \tilde{z}_1 - G_o\tilde{z}_2 \end{aligned} \quad (15)$$

which is accomplished by the control law  $\mu = (1/E)[V_d - R_i(z_1 - G_o V_d)]$ . For a detailed derivation, see [13]. In order to ensure a nonoscillatory asymptotically stable response, Theorem 1 has to be satisfied. The dissipative part of the mixed-potential of the closed-loop error system (15) is set to  $P_M^\mu(\tilde{z}) = (1/2)R_i\tilde{z}_1^2 + (1/2)G_o\tilde{z}_2^2$  and  $\Psi = 1$ .

<sup>4</sup>This means that for every admissible  $\mu = \mu^*$ , the converter exhibits a different (driving point) impedance.

Hence, by noting that  $\|\Psi\|^2 = 1$ , condition (11) leads to  $\|K_R\|^2 = (L/R_i^2 C)$  and, hence, we obtain

$$R_i = \frac{1}{1-\gamma} \sqrt{\frac{L}{C}} > \sqrt{\frac{L}{C}}$$

where  $\gamma$  is restricted to the interval  $0 < \gamma < 1$ . This places a lower bound on  $R_i$  in terms of the storage elements  $L$  and  $C$  only, regardless of the load  $G_o \geq 0$  and the control  $\mu$ . At this point, it is interesting to remark that the actual closed-loop system has a nice circuit-theoretic interpretation. To see this, consider the closed-loop system

$$\begin{aligned} L\dot{z}_1 &= \alpha V_d - R_i z_1 - z_2 \\ C\dot{z}_2 &= z_1 - G_o z_2 \end{aligned} \quad (16)$$

with  $\alpha = 1 + R_i G_o$ . Equation (16) can be interpreted as shown in Fig. 2. Notice that  $R_i$  indeed acts as a virtual series damping resistance.

#### B. Boost Converter

Let us next study the series PBC of the boost converter. For that, we aim at a closed-loop dissipation potential  $P_M^\mu(\tilde{z}) = (1/2)R_i\tilde{z}_1^2 + (1/2)G_o\tilde{z}_2^2$  yielding the closed-loop error dynamics

$$\begin{aligned} L\dot{\tilde{z}}_1 &= -R_i\tilde{z}_1 - (1-\mu)\tilde{z}_2 \\ C\dot{\tilde{z}}_2 &= (1-\mu)\tilde{z}_1 - G_o\tilde{z}_2. \end{aligned}$$



As already shown in [7], this is accomplished by the control law  $\mu = 1 - (1/\xi_2)[E - R_i(z_1 - z_1^*)]$ , where the desired inductor current is given by  $z_1^* = (G_o/E)V_d^2$  and  $\xi_2$  is the solution of the nonlinear differential equation  $C\dot{\xi}_2 = (z_1^*/\xi_2)[E - R_i(z_1 - z_1^*)] - G_o\xi_2$ . In a similar fashion as for the buck converter, a lower bound on  $R_i$  is found by applying Theorem 1. In this case,  $\Psi$  depends on the control  $\mu$ , i.e.,  $\Psi(\mu) = 1 - \mu$ , and the conditions for the theorem are straightforwardly checked to hold. Thus, Theorem 1 is satisfied if

$$R_i = \frac{1-\mu}{1-\gamma} \sqrt{\frac{L}{C}} > (1-\mu) \sqrt{\frac{L}{C}}.$$

Hence, for a given  $0 < \gamma < 1$ , if  $\mu$  is changing from one setpoint to another, we have for every  $\mu$  a different value for  $R_i$ , i.e.,  $R_i = R_i(\mu)$ .

### C. Robustness and Load Perturbations

Unfortunately, the series damping scheme is highly sensitive to unmodeled changes of the load. To see this, consider for example the series damping PBC regulated buck converter. Suppose that the load  $G_o$  is unknown, but bounded, i.e.,  $0 \leq G_o < \infty$ . Decompose the load into a nominal value  $G_n$  and a bounded uncertainty  $\Delta G_o$ , i.e.,  $G_o = G_n + \Delta G_o$ . The closed-loop dynamics (16) then change to

$$\begin{aligned} L\dot{z}_1 &= \alpha V_d - R_i z_1 - z_2 \\ C\dot{z}_2 &= z_1 - (G_n + \Delta G_o)z_2. \end{aligned} \quad (17)$$

with  $\alpha = 1 + R_i G_n$ , since we use the nominal value  $G_n$  in the control. Now, the equilibria of (17) are obtained as

$$z_1^* = (G_n + \Delta G_o)V_d \quad z_2^* = \frac{\alpha V_d}{1 + R_i(G_n + \Delta G_o)}$$

from which it is seen that if  $\Delta G_o \neq 0$ , the output capacitor voltage  $z_2$  will not converge to its desired value  $V_d$ . Similar arguments hold for the series damping controlled boost converter.

*Remark 4:* In order to deal with the load uncertainties, the controller can be extended with an adaptive mechanism to compensate for them [7]. A major disadvantage of this method is that the resulting controllers become quite involved, even for simple systems like the boost converter. Another problem that arises is how to tune the adaptive controllers as to ensure stability and nonoscillatory responses. A simple solution to prevent the use of an adaptive mechanism is presented in the following section.

## V. PARALLEL DAMPING PBC

In this section, we introduce the concept of parallel damping injection. As is done for the series damping injecting controllers, we again use the buck and boost type converters to illustrate the rationale of the approach. It is shown that this concept has some advantages in contrast to the series damping injection strategy, since it provides an easier solution to preserve the desired equilibrium in case of an unknown load. Moreover, it is shown that parallel damping injection also enables us to regulate a nonminimum phase circuit by measuring its nonminimum phase output only.

### A. Buck Converter

The parallel damping scheme is accomplished as follows. Instead of injecting damping in the undamped state  $z_1$ , we aim at voltage controlled resistors only, resulting in a dissipation potential of the closed-loop error system

$$P_M^\mu(\tilde{z}_2) = \frac{G_n}{2} \tilde{z}_2^2 + \frac{G_i}{2} \tilde{z}_2^2 \quad (18)$$

where we have assumed to have only knowledge of the nominal open-loop load. Following again the procedure of Section III, one possible controller that achieves the stabilization task can be derived as  $\mu = (1/E)(V_d - G_i L \tilde{z}_2)$ . (Notice that  $\tilde{z}_2$  can be obtained by measuring the capacitor current  $i_C = C\dot{\tilde{z}}_2$ .) In this case, the unperturbed (i.e.,  $G_o = G_n, \Delta G_o = 0$ ) closed-loop error dynamics satisfy

$$\begin{aligned} L\dot{\tilde{z}}_1 &= -\tilde{z}_2 \\ C\dot{\tilde{z}}_2 &= \tilde{z}_1 - (G_n + G_i)\tilde{z}_2 \end{aligned}$$

and, hence, the tuning criterion for the injected damping  $G_i$  yields

$$G_i = \frac{1}{1-\gamma} \sqrt{\frac{C}{L}} - G_n. \quad (19)$$

Another way to obtain a lower bound on  $G_i$  is recently proposed in [3]. This method uses the impedance properties of the storage elements,  $L$  and  $C$ , to find a precise match with the load conductance  $G_n + G_i$ . For the parallel damping PBC regulated buck converter we need to consider the closed-loop error equation for the average capacitor voltage  $\tilde{z}_2$ , i.e.,

$$\ddot{\tilde{z}}_2 + \underbrace{\frac{G_n + G_i}{C}}_{2\beta\omega_o} \dot{\tilde{z}}_2 + \underbrace{\frac{1}{LC}}_{\omega_o^2} \tilde{z}_2 = 0$$

where  $\omega_o$  is the resonance frequency and  $\beta$  is the damping factor. From classical control theory, we know [8] that in order to have a perfect damping,  $\beta$  has to satisfy  $\beta = 1$  (critical damping). This is accomplished if

$$\left( \frac{G_n + G_i}{C} \right)^2 = 4\omega_o^2$$

and thus by letting

$$G_i = \frac{1}{Z_c} - G_n = \frac{1 - G_n Z_c}{Z_c} \quad (20)$$

where  $Z_c \triangleq (1/2)\sqrt{L/C}$  is referred to as the characteristic impedance of the circuit. If now  $\beta > 1$ , then still a nonoscillatory response is guaranteed as long as the injected damping satisfies  $G_i > (1 - G_n Z_c)/Z_c$ . However, for values  $\beta \gg 1$ , the response will become sluggish.

Notice that if  $G_n > Z_c^{-1}$ , the necessary injected damping to satisfy (20) becomes negative, i.e.,  $G_i < 0$ . Strictly speaking, the controller then provides energy to the circuit and loses its passivity properties. On the other hand, consider the time-derivative of  $H^*(\tilde{z})$  along the trajectories of the closed-loop error dynamics  $\dot{H}_M^*(\tilde{z}) = -(G_n + G_i)\tilde{z}_2^2 \leq 0$ .

It is easily checked from (20) that the closed-loop dissipation  $G_n + G_i$  remains nonnegative for all  $Z_c > 0$ , even if (20) leads to  $G_i < 0$ , and thus passivity of the closed-loop system is preserved. Hence, by using Lyapunov theory and La Salle's invariance principle, see, e.g., [10], one can easily proof that the proposed controller indeed stabilizes the closed-loop dynamics of the system for all  $0 \leq |G_i| < G_n < \infty$ . For this reason, we refer to a PBC based on the parallel damping strategy as a *passivity-preserving controller* (PPC).

*Remark 5:* It is easily checked that for  $\gamma = (1/2)$ , the BM criterion (19) precisely coincides with the characteristic impedance matching criterion (20).

Now, that we have two criteria to tune the control parameter  $G_i$  for a known and constant nominal load, let us next study the case that  $\Delta G_o \neq 0$ . The actual capacitor voltage dynamics is then obtained by substitution of the control law into (4). After solving for  $z_2$ , we obtain

$$\ddot{z}_2 + \frac{G_o + G_i}{C} \dot{z}_2 + \frac{1}{LC} z_2 = \frac{1}{LC} V_d. \quad (21)$$

As for the series damping case, it is worth noting that (21) implies that there is a (virtual) resistor  $G_i$  connected in parallel with the capacitor and the load resistor. For that reason, we may refer to  $G_i$  as a *virtual parallel damping resistor*.

Considering (21), it is directly seen that for  $t \rightarrow \infty, z_2 \rightarrow V_d$ , independent of  $G_o = G_n + \Delta G_o$ . In other words, unlike for the series damping controller of Section IV, the equilibrium output voltage of the parallel damping controlled buck converter is independent of the load resistor. Summarizing, we have proved the following proposition.

*Proposition 1:* Consider the system (4) in closed-loop with the parallel damping PBC  $\mu = (1/E)(V_d - G_i L \dot{z}_2)$ . Given a desired value  $V_d \leq E$  for the average capacitor voltage  $z_2$ , the controller globally exponentially stabilizes the current and voltage trajectories of the average PWM model (4), where  $x$  is replaced by  $z$ , toward  $z_2^* = V_d$  and  $z_1^* = G_o V_d$ , for every  $0 \leq G_o < \infty$ .

## B. Boost Converter

Similar to the buck converter case, the regulation scheme based on parallel damping injection for the boost converter can be summarized as follows. First we make a copy of the circuit dynamics (5), with  $R_s = 0$ , in terms of  $\xi$  and, in contrast to the series damping philosophy used in [7] and [13], we inject the damping at the voltage coordinate, like in (18). Hence, in order to obtain an internally stable controller we set  $\xi_1 = z_1^*$  and solve for the control  $\mu$ . This results in the PBC

$$\mu = 1 - \frac{E}{\xi_2} \quad (22)$$

where  $\xi_2$ , for  $\xi_2(0) > 0$ , is the solution of the nonlinear differential equation

$$C \dot{\xi}_2 = G_n \frac{V_d^2}{\xi_2} - (G_n + G_i(\mu)) \xi_2 + G_i(\mu) z_2. \quad (23)$$

Notice that now the only signal used for feedback is the *non-minimum phase* output capacitor voltage  $z_2$  (while for a series

damping PBC one needs to measure the *minimum phase* inductor current  $z_1$ ). Furthermore, let the injected damping  $G_i(\mu)$  satisfy the BM criterion stemming from (12)

$$G_i(\mu) = \frac{1 - \mu}{1 - \gamma} \sqrt{\frac{C}{L}} - G_n.$$

*Proposition 2:* Given a desired value  $E \leq V_d < \infty$  for the average capacitor voltage  $z_2$ , the controller (22), together with (23), locally exponentially stabilizes the current and voltage trajectories of the average PWM model (5) toward the equilibrium  $z_2^* = V_d$  and  $z_1^* = G_o V_d^2 / E$ , for every  $0 \leq G_o < \infty$ .

*Proof:* We start by showing that (22)–(23) is a suitable controller for the stabilization task with respect to the internal stability, i.e., although we only measure the nonminimum phase output variable  $z_2$ , the zero-dynamics of the controller remain stable. For that, we proceed by eliminating  $\xi_2$  from (23) by using (22). Then, after some algebraic manipulations we obtain

$$\dot{\mu} = G_n \frac{V_d^2}{C E^2} (1 - \mu)^3 + G_i(\mu) \frac{z_2}{C E} (1 - \mu)^2 - \frac{G_n + G_i(\mu)}{C} (1 - \mu). \quad (24)$$

The zero-dynamics are obtained by letting  $z_2$  coincide with its desired value in (24), that is  $z_2 = V_d$ . The phase-plane diagram of (24), depicted in Fig. 3, shows that  $\mu = \mu^* = 1 - E/V_d$  for all  $E \leq V_d < \infty$  is a locally stable equilibrium point, while  $\mu = 1 (V_d = \infty)$  is unstable. Instability of  $\mu = 1$  corresponds to the fact that if the switch is in the ON-position for too long, the current through the inductor increases until the converter blows up. We conclude that the controller, although based on measuring the *nonminimum phase* output voltage only, is feasible for all  $\mu$  in the range  $0 \leq \mu < 1$ .

The proof of local exponential stability for the PBC based parallel damping follows from Theorem 2 and by the fact that  $\Psi(\mu)$  is Lipschitz. Furthermore, the proof that  $z_2 = V_d$  is preserved in the equilibrium in spite of uncertainties in the load is easily seen by considering the equilibria of (5) and (23). ■

The design of the passivity-preserving control algorithms based on either series or parallel damping injection scheme is carried out for the average PWM models of the buck and boost type converters. If the constant PWM switching frequency is chosen sufficiently high these models will capture the essential dynamic behavior of the converters, and, as a result, the controllers are well defined. Although we have only treated two simple examples, the design and tuning methodology is also applicable to a broad class of other power electronic circuits, as long as the average PWM model of such circuit has a BM structure.

## VI. SIMULATION RESULTS

The general closed-loop representation of the parallel damping PBC design philosophy regulating a real power converter, i.e., with the actual current and voltage states instead of the average ones, is depicted in Fig. 4. Here  $F_s = T_s^{-1}$  denotes the PWM frequency and  $G_i(\mu)$  is a matrix representing the

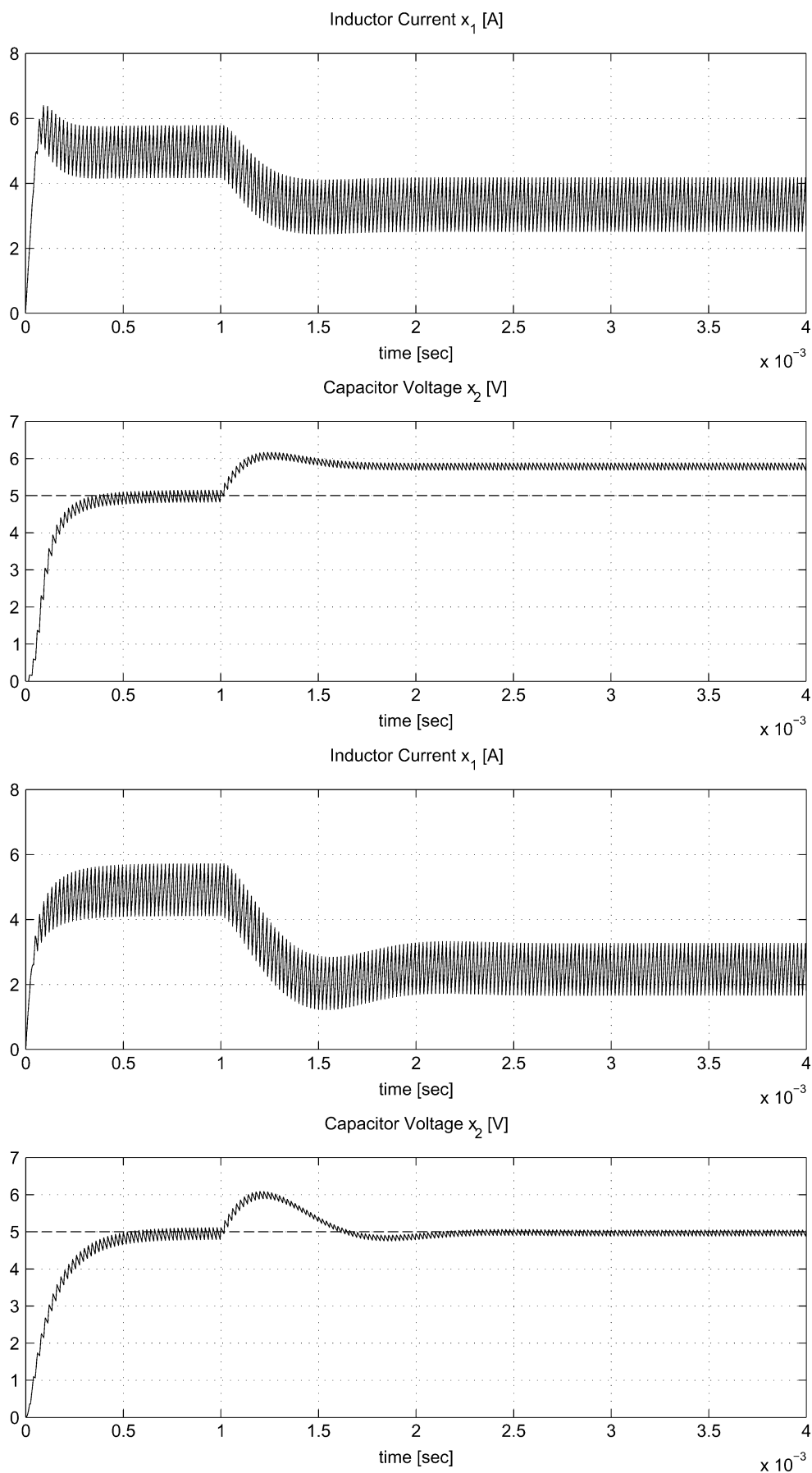


Fig. 8. Closed-loop response for load perturbations  $\Delta G_o = +(1/2)G_n$ : series damping PBC (top); parallel damping PBC (bottom).

virtual injected parallel damping conductances. Notice that in principle only the capacitor voltages used for feedback have to be measured, except in some situations where it is also possible to design a controller based on capacitor current feedback as is done here for the buck type converter. If the converter has more than one switch, say  $m$  switches, then  $\mu$  represents a  $m$ -dimensional column vector.

In the remainder of this section we test and compare both series and parallel tuning criteria using SIMULINK. We will use a boost converter with the discrete values for the switch. This means that for the series damping injection scheme the only signal used for feedback is the “real” inductor current  $x_1 \geq 0$ , and for the parallel damping scheme we only use the “real” capacitor voltage  $x_2 \geq 0$ . The design parameters of the Boost converter are chosen as follows:  $E = 1\text{V}$ ,  $L = 10\ \mu\text{H}$ ,  $C = 50\ \mu\text{F}$ ,  $G_n = (1/30)\Omega^{-1}$ , and the PWM switching frequency is set to 50 kHz. The initial conditions are set to  $x_1(0) = x_2(0) = 0$  and  $\xi_2(0) = 1$ .

In Fig. 5, a typical open-loop start-up response is shown for the boost converter. The response shows a large overshoot and is highly oscillatory. In Figs. 6 and 7, the responses of the output capacitor voltage are depicted for different setpoints. We observe that for both schemes the controller, with  $\gamma = (1/2)$ , rapidly stabilizes the capacitor voltages without any overshoot and oscillations. However, the series damping of Fig. 6 does not reach the desired voltage  $x_2 = V_d$  (dashed line), while the parallel scheme of Fig. 7 reaches the setpoints within 2% accuracy. The steady state error caused by the series damping PBC is due to the fact that the ripple in the inductor current is usually much higher than the ripple in the output voltage. A better accuracy could be obtained by increasing the PWM frequency or by selecting a larger inductor. Notice that the ‘undershoot’ in the capacitor voltage is caused by the nonminimum phase nature of the converter.

Furthermore, Fig. 8 shows the closed-loop response for load perturbations. These perturbations are set to  $\Delta G_o = +(1/2)G_n\Omega^{-1}$ , while both schemes are adjusted to a nominal capacitor voltage of 5 V. As expected from the theory, the parallel damping scheme rapidly manages to restore the capacitor voltage to its nominal value, while the series damping scheme does not manage to restore but forces the closed-loop to deviate from the desired voltage.

## VII. CONCLUSION

In this paper, the passivity-based controller design procedure for EL systems in the context of power converters is rewritten in terms of the BM equations. Besides the stability theorems proposed in this paper, the advantage of this setting is that the states to be used for feedback are directly in terms of physically measurable quantities, i.e., currents and voltages. This in contrast to Lagrangian or Hamiltonian systems, where the coordinates are usually the charges and the fluxes, which in most cases can not be measured directly. Additionally, the assignment of parallel damping does in general not involve the use of current sensors

but only needs the measurements of the voltages. Based on the studied examples, it appears that a major advantage of parallel damping in comparison with series damping injection is that it robustifies the closed-loop system in the sense that it does not require adaptive extensions in case the load is unknown or varying. Additionally, the idea of parallel damping injection provides a method to control nonminimum phase circuits based on the corresponding nonminimum phase output(s) only.

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